DRAWING ACCURATE AND PRECICE MAPS OF THE DEFLECTION OF THE VERTICAL COMPONENTS F0R EGYPT BASED ON HETROGENEOUS GRAVITY FILED DATA

By

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1- Abstract:

Modeling the deflection of the vertical components in certain area, have always vital role whenever precise geodetic computation is needed. Therefore the objective of the current thesis was to draw accurate and precise maps of the two components of the deflection of the vertical from the computed values of the same, by the least-squares collocation technique (LSC), at a grid of 5'x5' over the whole territory of Egypt, based on all the old and the recent available heterogeneous geodetic data related to the gravity field in the considered area.

Before appling the computational technique i.e, (LSC) to the available data, which are refered to different datums, these data where reduced to one and the same datum, the reduction was done by using several groupes of transformation parameters beetween the datum of a specific collection of data and the unifed datum (WGS84). The reduced rough data were filtered and then subjected to a process of smoothness before the predection step. The well-known remove-restore technique was used for achiving the maximum smoothness of the data, by the aid of an appropriate high degree global harmonic model, tailored to the local Egyptian terrestrial data. A digital terrain model of Egypt was also used for the same purpose. The final results have shown a rather accurate and precise maps of both predicted components of the deflection of the vertical over the intire territory of Egypt ($22°N \leq \varphi \leq 32°N$; $25°E \leq \lambda \leq 36°E$).

2- Introduction

In all precise geodetic works the determination of the values of the deflection of the vertical components is consider of an ultimate importance. These components, serves also in determining and refining the elements of the earth's gravity field in local areas as well as regional ones, thus it can be viewed as a necessary transition stage between the physical nature of the geodetic terrestrial observations and the geometrical nature of the adopted geodetic datum, realised

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by a certain ellipsoid (Heiskanen. and Moritz, 1967, Lachapelle, and Tscherning, 1978, Nassar, 1987).

Geodetc measurements (horizontal angles, zenith distances, astronomic latitudes, longetudes and azimuths) are observed on the topographic surface of the earth with respect to the local gravity vertical (plumbline direction), while geodetic computations are related to the normal to the adopted computational ellepsoid, the difference between the direction of the vertical and that of the normal (the deflction of the vertical) has an influnce on the geodetic computations. Globally, the magnitude of the deflection angle could reach one minute of arc, relative to the WGS84 reference ellipsoid (Collier and Croft, 1997). The neglection of that influnce causes a significant errors in position computations of any geodetic control network, which may reach several meters, or even in some cases,of large networks, tens of meters in the computed geodetic positions (Nssar, 1984).

Modeling and refining the geoidal surface, as a representative of the geopotential field in certain areas, is a very important task in geodetic practice. As the deflection of the vertical components represent the angular miss-alignments of the geoid with respect to the adopted reference surface, therefore, they constituted the oldest source of data for some methods of determining the geoidal surface. The deflections of the vertical can be related to any reference surface , usually to an ellipsoid of revolution, either local or geocentric. By these deflections, the geoid relative to the same reference surface is obtained, which realizes a relative or absolute geodetic datum. The accuracy obtained from such methods depends mainly on the amount and distiribution of the deflections data.

The deflection of the vertical components are computed at certain points by comparing the observed astronomic values, which are very few, and the computed geodetic values of the coordinates at the same points. At other points, were there exist no astronomic observations, the deflection of the vertical components were traditionally obtained by interpolation. However, since in most cases, the number of such astronomic stations are very few, exiset only at certain triangulation stations of the firest order (Lplace's stations), and the densification of such observations is consider not practical, because they are cumbersome, time consuming and rather expensive, therfore the interpolation of such values are seldom done.

There exist another methods of computing the deflection of the vertical components, e.g gravimetrically by solving what is called the boundary value problem (BVP) of physical geodesy specified for those components through the use of Vening Minesiz equations. However, this classical technique depends upon the gravity anomaly data, which suffers from the inadequate and inhomogeneous distribution, that leads obviously to insufficient accuracy of the obtained results.

Therfore, the objective of the current work is to consider the above mentioned remarkes and try determining the deflection of the vertical components in the whole terretory of Egypt using all the available geodetic and gravemetric data in the considered area.

Modelling any geodetic element related to the earth's gravity field demands an accurate technique for computation wich is compatable with the accuracy of the modern geodesy which is now going to reach 1part in 10^8 in spacial position determination. The least-squares collocation technique is recently considered as the most accurate, general and flixable trend in such computation. The principle of this algorithm is so flixable that it was generalised, in order that the input and the output items should not necessarly be of the same type, but could be of any heterogeneous nature, i.e the input items could be composed of any elements pertaining to any irrigular data configuration, while the solution (output), could be any desired type of the anomalous signals, related to the same field (Moritz, 1973a, 1980, Tscherning, 1982, 1987). The LSC estimates signals which have minimum variance among all other solutions, it also could estimate the required signal at any optional point, distriputed regularly or otherwise. Therfore, the LSC algorithm will be the routine technique used for modeling the dflection of the vertical components in the current invstigation.

Before appling the computational technique i.e, (LSC) to the available data, which are refered to different datums, these data should be reduced to one and the same datum, this was done by using several groupes of transformation parameters each of these groupes was used to reduce a specific collection of data related to certain datum to the unifed datum (WGS84). The reduced rough data were then filtered, and those found subjected to gross errors were excloded while the remained ones have been smoothed as greatly as possiple before computation.

The well-known remove-restore technique was used for achiving the last purpose, of maximally smoothing the remained data, , in order that these data and hence the covariance function, representing the same field, to be of purely random nature (Moritz, 1973a,b, 1978), as a pre-request by the collocation method. This was done by the aid of an appropriate high degree global harmonic model, tailored to the local Egyptian terrestrial data, where tailoring a model means that, a global high degree harmonic model is forced in some way to match the local data well . A digital terrain model of Egypt was also used for the same purpose, the author of this investigation has shared in computing both the tailored high degree global harmonic model and the digital terrain model of Egypt used in this study (Amin et. al 2003 a, b).

The produced residual gravity anomalies, after the last step, were used to determine what is called the empirical covariance function, that represents the local gravitational field in Egypt. The parameters of this function were applied in a *least-squares procedure to determine the best fitted analytical covariance function to it. The signals (residual data) remained at the data points, together with the analytical covariance function were then used in the prediction process, utilizing the law of variance-covariance propagation, via the LSC method to estimate the signals of the required components at the computation points. Afterwards, the effects of the removed parts of the data were added back (restored) to the previously predicted signals (residual values), at the same computation points, to get the final results. Thus, reaching the main goal of this research of computing the accurate and precise values of the deflection of the vertical components, at the nodes of 5'x5' grid for the whole territory of Egypt , which were then used to draw four maps representing these components as well as their precetions .*

*3- **Data preparation, reduction and validation**.*

The available heterogeneous gravity field data used in this work were primarily include free air gravity anomalies, gravity disturbance, geoidal heights, computed at GPS stations combined with elevations (from spirit levelling) and the deflection of the vertical components in both the meridian and prime-vertical directions at some stations of the first order triangulation network. These data were optained from the raw material of the $1^{st}$and 2^{nd} order gravity net observations, astronomic and geodetic co-ordenates of the primary horizontal control network, levilling heights and GPS satellite observations. A lot of time and great efforts had been exerted to collect the documntation of these huge amount of the Egyptian raw geodetic materials of different types. Details about some of these collected materials can be found in many publications and reportes e.g, (Kamel and Nkhla, 1987, Nassar, et al; 1993, 2000, Dawod, 1998, ESA, 1988, 1995).

A considerable amount of new gravity and GPS data have also been available, thanks goes to the National Institute of Astronomy and Geophysical Research. Also, some new valueble GPS- positions combined with spirit levelling heights relevant to the Egyptian Civilian Aviation Authority (ECAA, 1998), were available.

The positions of the collected heterogeneous data points were originally related to different datums; e.g. some to the non-geocentric Helmert (1906) reference ellipsoid and some to the WGS72 datum while others to the geocentric datum WGS 84. However, in order to use these data all together, they must be standardized by relating them to a unified system. Hence, these different data types were all reduced to the same geocentric system, namely the WGS84 worled geodetic datum. The transformation of the horizontal coordinates given on any datum into the WGS84 system was simply performed, using the orthometric heights as an approximation to the relevant ellipsoidal heights, then applying the relevant set of transformation parameters in each case (Bolbol, et al., 1997, , El-Tokhey, 2000).

This showed to have a nglected effect on the transformed horizontal positions, as was previously concluded by (Abd-Elmotaal and El-Tokhey, 1997). Hence, after transforming the horizontal positions of all raw data into the WGS84 reference ellipsoid, these data were used to assess the anomalous heterogeneous gravity fieled data.

The above prepared heterogeneous gravity field data were grouped in five separate data files, according to their types. Each file was firestly checked to remove any duplicated data pointes from the file. To insure that the input data has a good quality, which is very essential for a reliable predicted features, these files were supjected to a validation process, that aimed to filter out any data element, which was suspected to suffer from gross errors, so that the remaining validated data were assumed to be contimenated only by random noise. In other words, the job of the validation procedure was to reject the data which lack a minimum level of guarantee and reliability. For this respect it was therefore necessary to compare the observed value to the predicted one, estimated by a powerful method.

The whole heterogeneous data were used in a simultaneous collocation prediction to estimate (predict) respective signals at the same data points. This has the effect of data filtration from noise and in this case, there would be nearly no effect of field roughness or data gaps (Tscherning, 1982, Sevilla et al., 1990). Then, one would reject a specific observation having gross errors, if the following condition were satisfied:

$$| S_{observed} - S_{predicted} | > 3.0 \, (\sigma^2_{observed} + \sigma^2_{predicted})^{1/2} \qquad (1)$$

In which the right- hand side is the Gaussian sum of the a-priori input noise and the a-posteriori error estimate. The same condition was also useful in filtering out very closely spaced stations, which would deteriorate the LSC solution.

The number of the collected data points of each type and the number of the filtered ones are listed in table (1),and their distribution is shown in figure (1).

Table (1): The raw and filtered data numbers and types

Item	Total data No.	Filt. data No.
Geoids(GPS-Lev.)	352	159
Gravity anomalies	1486	1290
Gravity disturbances	313	231
η (prime vert.-def.)	13	13
ξ (merid.-def.)	147	131

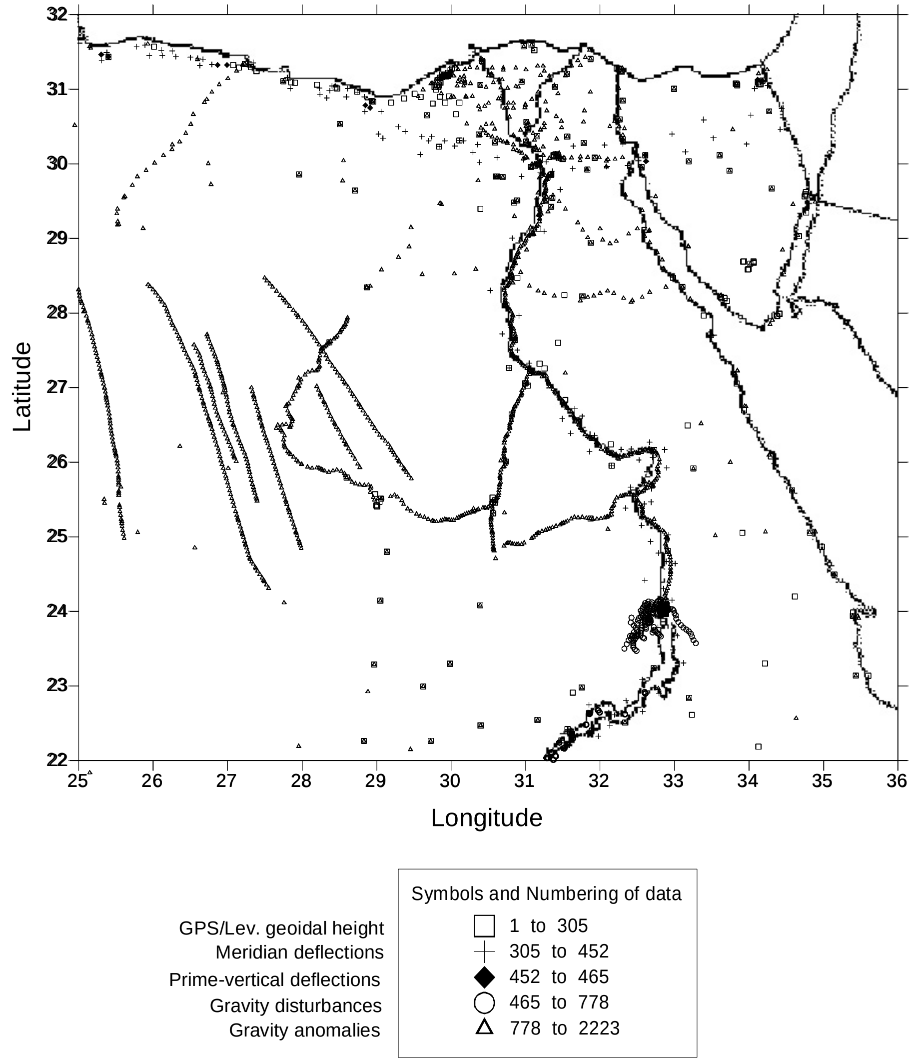

Figure (1): Distribution of the available scattered heterogeneous data related to the gravity field in Egypt

4- The geometrical signeficance of the deflection of the vertical

Since the astronomic quantities (φ',λ', α') are refered to the geoid (if we neglect the small correction due to the curvature of the plumb line), and the corresponding geodetic ones (φ, λ, α) are computed along the reference ellipsoid, the relationship between the two systems can then be materialized through the knowledge of the angle (θ) of the deflection of the vertical, which is defined as the angle subtended between the normal to the geoid (defined by the vertical at the observational point), and the normal from the same point to the used reference ellipsoid. This angle is conventionally decomposed into two components (ξ) , the meridian component, *positive northwards, and (η) , the east-west (prime-vertical) component, positive eastwards, such that for a spherical reference body, we have*

$$\xi = \varphi' - \varphi \qquad (2)$$

$$\eta = (\lambda' - \lambda) \cos \varphi, \qquad (3)$$

so that

$$\theta^2 = \xi^2 + \eta^2 . \qquad (4)$$

It should be emphasized that these relations are only valid, if and only if, the astronomical and the geodetic latitudes and longitudes are measured from the mutually parallel axes of rotations of the earth and of the reference ellipsoid (Molodensky, et al., 1962). Only in this case are the values of the astronomical and geodetic azimuths (α' & α) related to each other by the well known relation of a purely geometric character, written in a spherical approximation as;

$$\eta = (\alpha' - \alpha) \cot \varphi. \qquad (5)$$

Obviously, the formulation of the last equation makes it possible to link the differences between the astronomical and geodetic longitudes with differences of astronomical and geodetic azimuths, through the famous Laplace's equation, written in its simple spherical form as;

$$(\alpha' - \alpha) - (\lambda' - \lambda) \sin \varphi = 0. \qquad (6)$$

The ξ component is obviously known at all triangulation points, where the astronomic latitudes have been measured. The triangulation points where simultaneos determination of longitudes and azimuths are accomplished are called " Laplace stations", (Bomford, 1971).

The Laplace equation has nothing to do with the real deflections of the vertical, it only gives a measure of the accuracy of the difference ($\alpha' - \alpha$) and ($\lambda' - \lambda$). This equation is not exactly satisfied by the measured quantities owing to unavoidable errors of astronomical observations and errors in geodetically computed longitudes and azimuth, thus leave a remainder, the "Laplace discrepancy" given as a closure error "w" of the equation:

$$(\alpha' - \alpha) - (\lambda' - \lambda) \sin \varphi = w. \qquad (7)$$

Consequently, this error is caused, not by any deflections of the vertical but solely by errors in the observed and computed longitude and azimuth, as mentioned before, therefore, it may serve as a condition used in adjustement procedure of triangulation nets, see, e.g (Pick et al. , 1973, Wolf, 1967, 1982).

For the purpose of establishing a national geodetic system, a control point is to be chosen as the initial point, used as the origin of the geodetic coordinate system, where the astronomic quantities ($\varphi_0', \lambda_0', \alpha_0'$) should be measured as accurately as possible. These astronomic quantities are usally adopted, as the geodetic ones (φ_0 , λ_0 , α_0), to relate the initial point to the geodetic datum. From these values we proceed to compute the geodetic coordinates of the rest of the triangulation points along the chosen referance ellipsoid. In this way, the two surfaces to which both the astronomic and geodetic quantities are reffered i.e the geoid and ellipsoid surfaces respectively, are considered paralle at the initial point ,which implies that $\xi_0 = \eta_0 = 0$. furthermore, it is sometimes assumed that the geoidal height N_0 at the initial point is also equal to zero, in this case the two surfaces are thus considered to be tangent at the same point. In general, however, the two surfaces are neither parallel nor tangent. Thus, to convert these astronomical quantities at the initial point correctly from the geoid to the adopted ellipsoid , the ξ_0 and η_0 components should be computed precisly then their effect on the computed coordinates should be removed. Furtermore ,it has been shown by different authors that the nglection or the non-regorous treatment of the reduction of the terrestrial geodetic observations, or even the inaccurate determination of the geoidal shape, cause systematic distortions of the terrestrial networks;which can not be ignored for large triangulation chains (Zakatov,1962, Meissl, 1973).

5- The computational stepes

The remove-restore technique has become a standard tool in all modern regional and local gravity field elements determination. In principle, any a priori known signal features are firstly removed from all used input data, prior to the prediction process, and then appropriately added back (restored) to the predicted features. After the removal of those effects, the resulting residual data could be treated practically as centered data with possibly minimum signal mean and standard deviation. The removal usually involves the low frequency global features, implied by an appropriate high- resolution harmonic model of global geopotential. Moreover, a suitable high-resolution digital terrain (elevation) model DTM (DEM), is usually used to account for the local topographic effects, (Tscherning,1982a,b). These low-pass and high-pass filtering effects, respectively, result in smooth residual data that would produce the most accurate and precise predicted signals.

5-1 ***The tailored model EGM96EGCT, as the most appropriate model, used for recovering low frequency features of the gravity field in Egypt***

The fact that a harmonic model would be reliable only in regions with terrestrial gravity data contribution to it, becomes well known (Hanafy, 1993, Smith and Milbert,1997). If however, the removed harmonic model was not supplied with local data from the region under investigation, a very crude smoothing effect would be expected, since the model would only succeed to filter out a spectral content, which is comparable with the relevant satellite only information, which have a very low frequency nature and does not provide a suitable smoothing effect. Hence, a residual long wavelength error would be committed during its removal and restoration in a local (or regional) gravity field modeling. This, in turn, would decrease the accuracy of the estimated signals, beyand the current geodetic demandes.

It has been proven by different authors in many investigations, concerning the Egyptian situation that no local terrestrial gravity data has been introduced to the global harmonic analysis solutions for any of the old or even the newly available global geopotential models (El-Tokhey, 1995, Amin, 2002, Hassouna, 2003). The limited benefit of using such harmonic models, as global reference fields, should therfore be expected. This reality led the author with other colligues to solve the problem differently, instead of waiting that the secreat local Egyptian gravity field data should be supplied and merged into the solutions of such global harmonic models, which is doubtful, the local data was used to compute corrections, to be aded to the harmonic coefficients of such model, this operation made the newly obtained (tailored) model fit well the local Egyptian region (Amin et al., 2002).

Since the EGM96 global harmonic model represents the most recent and accurate high degree geopotential model (Lemoine et al., 1996) that is completely released to the geodetic community, hence in a previous work by the author and two of her collegues, this model was tailored to Egypt, using the LSC method, based on the local Egyptian heterogeneous data. The considered tailored model was called EGM96EGCT and was estimated up to degree and order 599. The reliability of the tailored model to recover the low-medium spectral features over Egypt in an efficient manner has been verified, (Amin et al., 2002, 2003b). The tailored model was therefore, used for removing the low frequency global features from all the used data according to the following relations:

$$N_{Model} = (GM/r\gamma) \sum_{n=2}^{Nmax} (a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\psi) \qquad (8)$$

$$\Delta g_{Model} = (GM/r^2)\sum_{n=2}^{Nmax}(n-1)(a/r)^n \sum_{m=0}^{n}(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\psi) \quad (9)$$

$$\delta g_{Model} = (GM/r^2)\sum_{n=2}^{Nmax}(n+1)(a/r)^n \sum_{m=0}^{n}(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\psi) \quad (10)$$

$$\xi_{Model} = -(GM/r^2\gamma)\sum_{n=2}^{Nmax}(a/r)^n \sum_{m=0}^{n}(\bar{C}^*_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\, d\bar{P}_{nm}(\sin\psi)/d\psi \quad (11)$$

$$\eta_{Model} = -(GM/r^2\gamma\cos\psi)\sum_{n=2}^{Nmax}(a/r)^n \sum_{m=0}^{n} m(\bar{C}^*_{nm}(-\sin m\lambda) + \bar{S}_{nm}\cos m\lambda)\bar{P}_{nm}(\sin\psi) \quad (12)$$

with the zero and first degree terms taken equal to zero, where

$N, \Delta g, \delta g$ *the geoidal height, gravity anomaly, and gravity disturbance*

ξ, η *the deflection of the vertical components in the meridian and the prime vertical directions, respectively,*

ψ *the geocentric latitude,*

λ *the geodetic longitude,*

r *the geocentric radius to the geoid,*

γ *the normal gravity implied by the reference ellipsoid,*

GM *the product of the Earth mass by the gravitational constant,*

a *the equatorial radius,*

$\bar{C}^*_{nm}$ *the fully normalized spherical harmonic C-coefficients of degree n and order m, reduced for the even zonal harmonics of the reference ellipsoid,*

$\bar{S}_{nm}$ *the fully normalized spherical harmonic S-coefficients of degree and order m,*

$\bar{P}_{nm}(\sin\psi)$ *the fully normalized associated Legendre function of degree n and order m.*

N_{max} *the maximum degree of the used harmonic model, (in our case is 599).*

The square of any of the above expansions, at a specific degree, yields a relevant positive real number, which is referred to as the degree variance. This is evident from the orthogonality among the coefficients on one hand and among the surface harmonic functions on the other hand (Heiskanen and Moritz, 1967). The (positive) degree variance expresses how much signal power (content) is *implied by all the coefficients belonging to a specific degree, in a global sense. It is usually referred to as the power spectrum. Hence, the variation of power spectra with the degree describes in a practical way the rate of decay of the anomalous signal as the degree increases. The gravity anomaly spectra (degree variances) is, given for example (in spherical approximation) by,*

$$(\sigma^2_n)_{\Delta g} = (G')^2 .(n-1)^2 . \sum_{m=0}^{n} (\bar{C}^{*2}_{nm} + \bar{S}^2_{nm}) \qquad (13)$$

where G' is the mean gravity. If the harmonic coefficients in the above formula are replaced with their error estimates, σ_{C^*nm} & σ_{Snm}, *one obtains the so-called error degree variances (error spectra). Thus, one obtains the gravity anomaly error degree variance as follows*

$$(\sigma_e{}^2{}_n)_{\Delta g} = (G')^2 .(n-1)^2 . \sum_{m=0}^{n} (\sigma^2_{C^*nm} + \sigma^2_{Snm}) \qquad (14)$$

The (positive) error degree variance expresses how much signal power error of a given anomalous quantity exists, in a global sense, for all the coefficients of a specific degree. In general, the error degree variances and degree variances are very useful in covariance function modeling (Foresberg, and Tscherning, 1981, Tscherning, 1993, Gruber, et al. 1997).

5-2 The digital elevation model (DEM) used for recovering the high frequency features of the gravity field in Egypt

In another previous thesis made by the same authors (Amin et. al, 2003a), a detailed 5'x5' digital elevation model for Egypt was computed by collocation, based on the available local height data and the global high-resolution topographic harmonic model GTM3a. This DEM model was used in this work to filter out the high frequency features, reflected by the local topography, from the input data elements, where its contribution, related for example, to the gravity anomalies and the geoidal heights quantities were determined according to the following expressions

$$\Delta g_{DEM} = 2\pi G\rho(h-h_{ref}) - T_c$$

$$= 2\pi G\rho(h-h_{ref}) - (G\rho R^2 /2)(\Delta\varphi.\Delta\lambda)\Sigma((h'-h)^2 / l^3)) \qquad (15)$$

$$N_{DEM} = -(2\pi G\rho/\gamma)(h-h_{ref})^2 - (G\rho R^2/6 \gamma))(\Delta\varphi.\Delta\lambda) \Sigma((h'^3-h^3) / l^3) \qquad (16)$$

where Tc is the classical terrain correction with respect to the Bouguer plate. The first term is the Bouguer plate effect on the anomaly or geoid data.

h is the orthometric height of the computation point,

h' is the orthometric height of the running point,

G is the gravitational constant,

P is the mean crustal density,

h_{ref} is the relevant elevation of the average surface

l is the spatial distance between the computation and running points.

Table (2) through (6) show the statistics of the input and the residual, gravity anomaly data, gravity disturbances, geoidal heights, meridian and prime-vertical deflection components, respectively, based on the EGM96EGCT and the DEM). These tables show that the removal of the global and the local information had great smoothing effect on most of the used gravimetric data, in terms of the mean, the standard deviation and root mean square value. It is also clear that the smoothing effect of the DEM is rather small than that of the used geopotential harmonic model.

***Table (2): Statistics of original and residual gravity anomaly data* (units: mgals)**

Item	Mean	Std. Dev.	RMS	Min.	Max.
Free air gravity anomaly	-5.916	29.142	29.725	-78.234	144.623
DEM reduced gravity anomaly	-1.987	27.986	28.045	-75.571	121.144
Final(DEM+EGM96EGCT) residual gravity anomaly	2.128	15.918	16.053	-76.757	102.894

Table (3): Statistics of original and residual gravity disturbance data (units: mgals)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Free air gravity disturbance	-2.159	8.182	8.445	-23.389	18.032
DEM reduced gravity disturbance	0.597	8.434	8.437	-19.081	22.584
Final(DEM+EGM96EGT) residual gravity disturbance	-0.815	6.775	6.810	-19.021	17.858

Table (4): Statistics of original and residual geoidal height data (units: meters)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Geoidal height	13.645	3.598	14.108	5.884	21.140
DEM reduced geoidal height	13.729	3.612	14.194	6.143	21.176
Final(DEM+GM96EGC) residual geoidal height	0.043	0.853	0.851	-3.766	3.490

Table (5): Statistics of original and residual meridian deflection component data (ξ) (units: arc-seconds)

Item	*Mean*	*Std. Dev.*	*RMS*	*Min.*	*Max.*
ξ	*1.911*	*4.902*	*5.243*	*-9.545*	*13.740*
DEM reduced ξ	*1.810*	*4.480*	*4.816*	*-7.774*	*12.322*
Final(DEM+EGM96EGT) residual ξ	*0.577*	*3.198*	*3.237*	*-6.617*	*10.202*

Table (6): Statistics of original and residual prime-vertical deflection component data (η) (units: arc-seconds)

Item	*Mean*	*Std. Dev.*	*RMS*	*Min.*	*Max.*
η	*-0.330*	*5.500*	*5.294*	*-8.044*	*8.241*
DEM reduced η	*0.027*	*5.185*	*4.982*	*-6.724*	*7.685*
Final(DEM+EGM96EG) residual η	*-0.639*	*2.217*	*2.224*	*-6.106*	*2.373*

5-3 The determination of the empirical and the fitted analytical covariance function of the gravity field of Egypt

It is well known that the unique basic covariance function of the anomalous potential, which best describes the input anomalous features, has a very important role in gravity field modeling by collocation, because all input and output covariances are derived from such function. It is a function of the separation between the data points that describes the spatial variability of the local residual field under consideration. The collocation technique demands that the potential field, and hence the relevant covariance functions to be treated in the solution as being homogenous and isotropic, which requires that the handled data should be as smooth as possible, in order they behave purely random. This requirement has been achieved by the above-mentioned removal steps of the effect of the used tailored (HM) and the (DEM).

The covariance function of the anomalous potential in its general form is characterized by its ability to predict (or reproduce) any gravitational signal, once certain input elements of any type are given, via the relevant functional operators. In this sense, the potential covariance function K (p,q) is defined as the reproducing kernel function of the anomalous gravity field, from which any type of covariances can be easily derived from the known mutual relations among the anomalous potential and the other field elements given by the following relations:

$$\Delta g = - \partial T / \partial r - (2/r)T, \quad (17)$$
$$\delta g = - \partial T / \partial r, \quad (18)$$
$$N = (1/\gamma)T, \quad (19)$$
$$\xi = - (1/\gamma r) \partial T / \partial \varphi, \quad (20)$$
$$\eta = -(1/\gamma r \cos\varphi) \partial T / \partial \lambda . \quad (21)$$

In practice, the isotropic covariance function is estimated empirically in terms of the residual of the observable elements of the anomalous potential. In particular, discrete residual gravity anomalies on the sphere are most popular as input data to be utilized via a numerical integration, as follows

$$C(\psi) = (\Sigma \Delta g_{J.} \Delta G_{k.}) / u, \qquad (j \neq k) \quad (22)$$

Where u is the number of products taken at a given spherical distance ψ. *Thus, the empirical covariance function C at a spherical distance* ψ *is simply the product sum average of pairs of anomaly values, relevant to pairs of points (irrespective of the relevant mutual azimuths) having spacing* $\psi - \Delta\psi/2 \leq \psi' \leq \psi + \Delta\psi/2$, *where* ψ *and* $\Delta\psi$ *is suitably chosen interval and range respectively (Tscherning and Rapp, 1974). The main features of this function are the mean square of the (residual) anomaly data, called the variance (covariance at zero distance)* C_0, *the radius of curvature of the covariance function curve at the same point,* χ, *the correlation length* ξ, *which corresponds to a positive covariance value that is equal to half the variance, and the location of the first zero of the covariance function,* $\psi_{0.}$

Both the variance and the correlation length of the empirical function express the amount of smoothness of the input data, which is function of the accuracy and stableness of the solution. The smaller the values of the variance and the correlation length are, the smoother is the input data. In addition, a smaller data variance guarantees a smaller mean prediction error, while a small correlation length implies a well-conditioned auto-covariance matrix, C_{tt}, *of the data, and hence, a more stable LSC solution.*

As this function depends only on the spherical distance, ψ, *between pairs of data points, a 100* ψ *values were found enough to determine the parameters of the empirical function used in the current work, while both,* ψ *and* $\Delta\psi$ *increment were chosen to be 2 minutes of arc.*

The next step is the formulation of the model (analytical) covariance function that is best fitted to the empirical one in a least-squares sense. This model covariance is uniquely described through three parameters. These three parameters, denoted as c (a unitless scale factor) may be used as a measure of how well the used harmonic model fits the local data low frequency information, *A (in mgal ^{2}), implied by the residual data variance at H=0, and $R - R_b$ (in meters) are given first approximate values. On the other hand, the residual gravity anomaly empirical covariance values are input and treated as observations, then the adjustment is performed in an iterative manner until the convergence is reached, resulting in the final values of the three parameters and the point gravity anomaly variance at MSL as a by- product. The local isotropic anomaly covariance function model can be given as (Tscherning, 1993)*

$$C(P,Q) = C(r,r',\psi)$$

$$= \sum_{n=2}^{N_{max}} c.\sigma^2_{neEGM96EGCT}.(R_b^2/rr')^{n+2} P_n(\cos\psi) + \sum_{N_{max}}^{\infty} A.2).(n+24).(R_b^2/rr')^{n+2}.P_n(\cos\psi) \quad (23)$$

where

ψ	the spherical distance between the two points P and Q,
r	the geocentric radial distance of point $P \approx R+H_P$,
r'	the geocentric radial distance of point $Q \approx R+H_Q$,
R	the mean radius of the Earth, taken ≈ 6371 km,
R_b	the radius of the Bjerhammar's sphere,
$\sigma^2_{neEGM96EGCT}$	the n^{th} anomaly error degre variance based on EGM96EGCT coefficients'standard erro
c	a positive unitless scale factor,
A	a positive constant ($mgal^2$),
N_{max}	599 (max degree of EGM96EGCT),
H	orthometric height of the respective point.

The final values of the parameters were then used as input for the collocation process, in which the law of covariance propagation was executed to compute all the elements of the possible varieties of auto-covariances matrix between the observations, the auto- covariances matrix of the signal to be predicted and the cross-covariances matrix between the input observations and the unknown signals (Tscherning and Rapp, 1974). This was done, in terms of the functional types at the observation points and the relevant positions. These computed matrices are considered as the backbone of the collocation solution.

Figure (2) illustrates the input residual anomaly isotropic empirical covariance function, and its associated fitted analytical function.

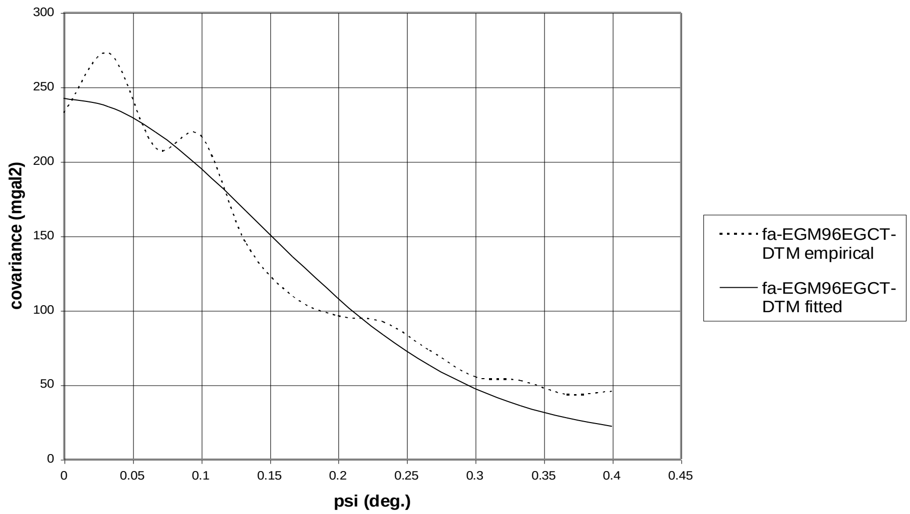

Figure (2): Empirical and fitted covariance function of the final residual data

5-*4 Computation and results throw the LSC method*

The respective residual gravitational elements (along with their noise), and the final relevant three parameters of the fitted covariance function were input into the LSC solution. The aimed results of the solution were the respective 5'x5' grid values for the two components of the deflection of the vertical along with the corresponding error estimates, computed by the well known LSC expressions as follows:

$$S = C_{st} (C_{tt} + E_{tt})^{-1} . \, l, \qquad (24)$$

$$E_{ss} = C_{ss} - C_{st}(C_{tt} + E_{tt})^{-1} . \, C_{st}^{T}, \qquad (25)$$

with

S — *the vector of estimated (predected), signals (residual elements),*
l — *the vector of residual (signal) input data,*
C_{st} — *the cross-covariance matrix between the signals S and the residual input data l,*
C_{tt} — *the variance- covariance matrix of the residual input data,*
E_{tt} — *the error variance-covariance matrix of the residual input data,*
E_{ss} — *the estimated error variance-covariance matrix of the estimated signals S,*
C_{ss} — *the variance- covariance matrix of the signals S.*

Then, the contributions of the two harmonic models, i.e. the tailored (HM) and (DEM), in the deflection components were then added back (restored) into the relevant residual deflections predicted at the grid nodes (computation points), in *order to obtain the respective 5'x5' complete (full) values of the deflection components, as well as their error estimates. Tables (7) and (8) outline the statistics of the various items of these solutions.*

Table (7): Statistics of the residual and full predicted meridian deflection components (ξ) (units: arc-seconds)

Item	*Mean*	*Std.Dev.*	*RMS*	*Min.*	*Max.*
Predicted residual ξ	*0.041*	*3.045*	*3.045*	*-29.943*	*25.314*
Full predicted ξ	*-1.011*	*6.584*	*6.661*	*-7.774*	*12.322*
ξ Ξerror Std. Dev.	*1.844*	*0.523*	*1.916*	*0.080*	*2.326*

Table(8): Statistics of the residual and full predicted prime-vertical deflection components(η) (units: arc-seconds)

Item	*Mean*	*Std.Dev.*	*RMS*	*Min.*	*Max.*
Predicted residual η	*0.073*	*3.139*	*3.140*	*-31.352*	*23.493*
Full predicted η	*0.637*	*6.826*	*6.855*	*-42.972*	*25.526*
η error standard deviation	*1.872*	*0.484*	*1.934*	*0.073*	*2.327*

Finally, Figure (3) through (6) show the contour maps of the deflection of the vertical components in the meridian and the prime vertical directions, respectively, as well as their error standard deviations.

*6- **Concluding remarkes***

From the attached tables and figures, it is clear that the computational prcedure used in the current investigation produced in general a rather accurate and precise results, with respect to the available data. However, in areas which suffer from ill distribution and larg gabs between data points, specially those with a relatively high altitude the standered deviations reach their maximum values for both deflection of the vertical components, as was expected, because of data deffeciency at these areas, which of course is a crutial matter for accurate and precise results and because at high altetudes the effect of topography has a significant influnce on the deflection of the vertical angle.
In general, these maps can be used satisfactory for interpolating the values of the deflectin of the the vertical components , whenever a primary control net work is considered, specially in areas which lack astronomical observations.

The values of these deflection components were computed at all the 1st order triangulation points of the national geodetic network of Egypt and are available under request for whome it may be concerned.

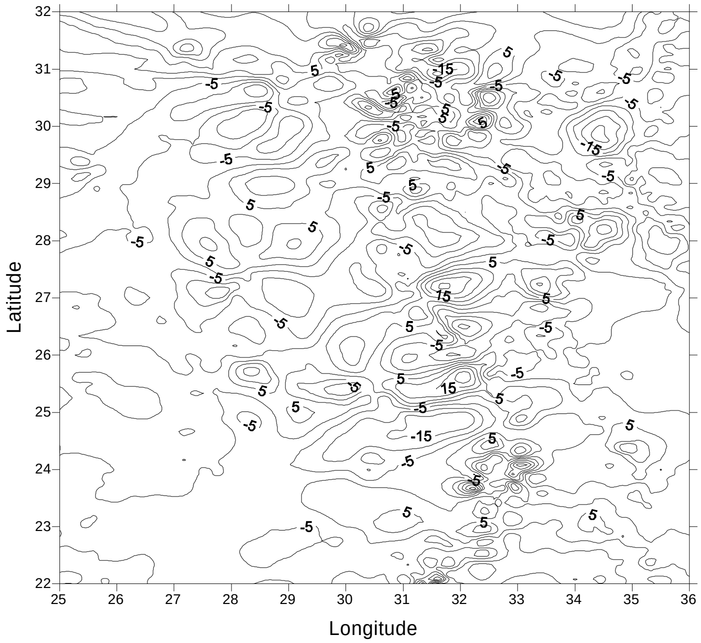

Figure (3): Contour map for the predicted meridian deflection components (ξ)
(Interval: 5 seconds of arc)

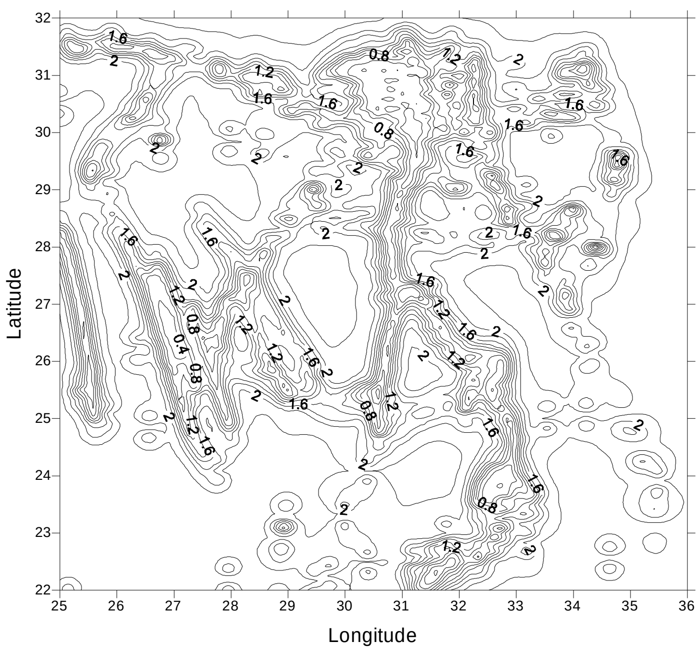

Figure (4) : Contour map for the error standard deviation of the meridian deflection components (ξ) (Interval : 0.2 arc-second)

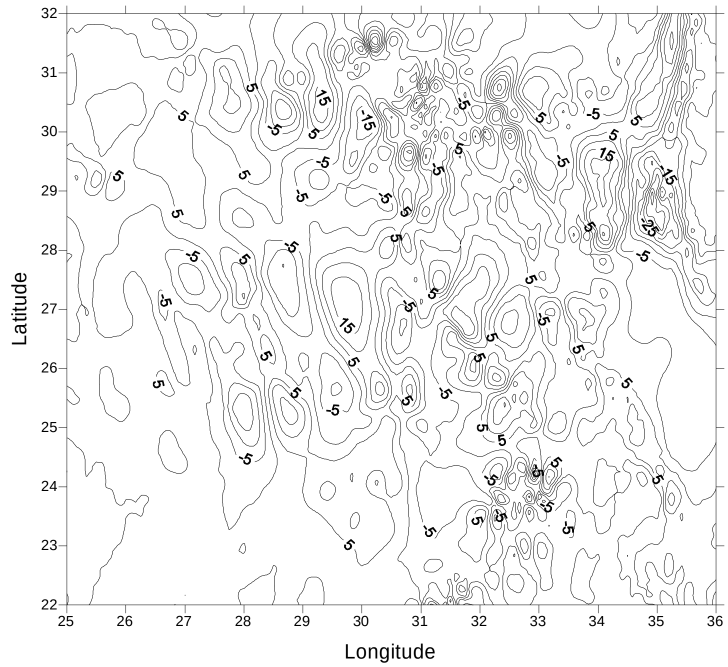

Figure (5): Contour map for the predicted prime-vertical deflection components (η)
(Interval: 5 seconds of arc)

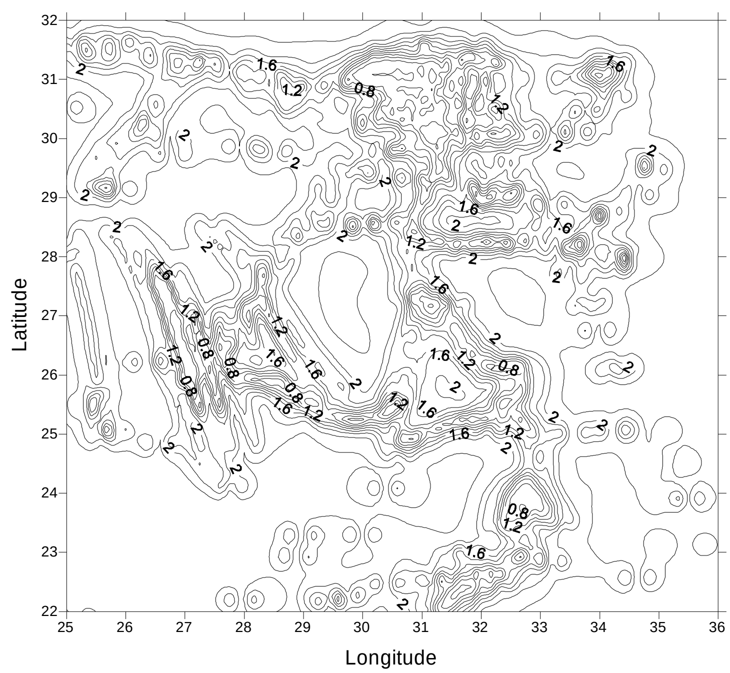

Figure (6): Contour map for the error standard deviation of the prime-vertical deflection components (η) (Interval: 0.2 arc-second)

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